Homotopy Methods in Topological Fixed and Periodic Points Theory

In mathematics, particularly in the field of topology, homotopy methods play a pivotal role in the study of fixed and periodic points. A fixed point of a map is a point that remains unchanged when the map is applied to it, while a periodic point is a point that returns to its original position after a certain number of iterations of the map.

Homotopy Methods in Topological Fixed and Periodic Points Theory (Topological Fixed Point Theory and Its Applications Book 3) by Jerzy Jezierski

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Homotopy methods provide a powerful toolkit for analyzing the existence and properties of fixed and periodic points. These methods involve deforming one map into another in a continuous way and studying the behavior of the fixed or periodic points along the deformation.

Topological Fixed Point Theorems

One of the most fundamental results in topological fixed point theory is the Brouwer fixed point theorem, which states that every continuous map from

a compact convex set to itself has at least one fixed point.

Homotopy methods can be used to prove the Brouwer fixed point theorem by constructing a homotopy from the original map to the identity map, which has a unique fixed point. Along the homotopy, the fixed point of the original map can be tracked, and it is shown that it must exist.

Other important topological fixed point theorems include the Schauder fixed point theorem and the Leray-Schauder theorem, which generalize the Brouwer fixed point theorem to more general spaces and maps.

Topological Periodic Point Theorems

In the study of dynamical systems, topological periodic point theorems play a crucial role. These theorems provide conditions for the existence of periodic points for continuous maps on topological spaces.

A well-known example is the Poincaré-Birkhoff theorem, which states that every continuous map of a compact metric space into itself has a periodic point.

Homotopy methods can be used to prove the Poincaré-Birkhoff theorem by constructing a homotopy from the original map to a map with a known periodic point. By using the properties of homotopy, it can be shown that the original map also has a periodic point.

Applications

Homotopy methods in topological fixed and periodic points theory have numerous applications in various branches of mathematics, including:

- Analysis: Homotopy methods are used to prove existence and uniqueness theorems for solutions of nonlinear equations and differential equations.
- Geometry: Homotopy methods are used to study the topology of manifolds and to prove important theorems such as the Poincaré conjecture.
- Algebraic topology: Homotopy methods are used to study the homology and cohomology of topological spaces.

Homotopy methods are a versatile and powerful tool in topological fixed and periodic points theory. They provide a deep understanding of the behavior of maps and their fixed and periodic points.

The applications of homotopy methods extend far beyond the realm of topology, and they continue to be a subject of active research in various branches of mathematics.

References

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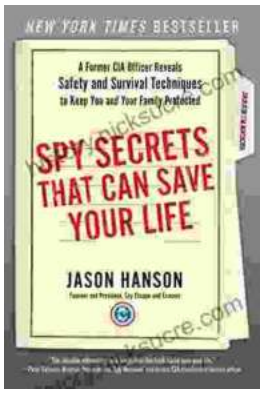

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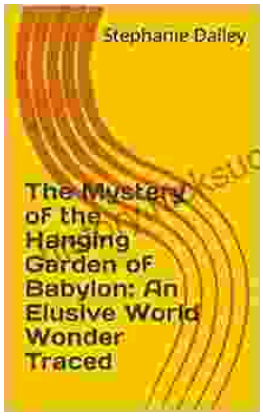

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